3.3.1. Conditional Semantic Problems:
Logical Equivalence

A. **Translate** each English sentence into the formal language, and build a **truth table** for that formal sentence. On the basis of that truth table, find a **simpler English sentence** that is **logically equivalent** to the original.

1. If Neko either won on Drastic Droids or didn't then she won on Drastic Droids.

2. If Rex cancelled class then he didn't cancel class.

3. It's not the case that if Dr. Slim is under arrest then he's not under arrest.

4. If Jake is playing pool, then if he's playing pool he's happy.

5. If Letitia can go clubbing if she bought Black Dye No. 1, then she bought Black Dye No. 1.

6. Jack will surf at Oahu – or, if not, at Teahupo'o.

7. Suki likes uni if she likes uni; otherwise she likes uni.

8. If Jezebel's gambling only if Jake is, then both Jezebel and Jake are gambling.

9. If Jezebel's gambling if Jake is, then both Jezebel and Jake are gambling.

(*For sentences 10 through 13: the simpler sentence won't appear as an earlier step in the truth table.*)

10. If Trixie's buying a hot tub only if she won the poker tournament, then she won the poker tournament.

11. If Barbie's going mountain climbing in either Thailand or Tibet, she's going mountain climbing in Tibet.

12. If we’re having truffles, we aren’t having *both* truffles *and* grog.
    (*Try to find a simpler conditional*.)

13. If Suki passed the exam, she did so without studying.
    (*Try to find a simpler conditional*.)

B. For each trio of sentences, build **truth tables** or **truth trees** to show which two sentences are **logically equivalent**. (For each trio, one of the sentences is *not* equivalent to the other two.)

1a. $((P \rightarrow Q) \rightarrow P)$
1b. $((P \rightarrow Q) \rightarrow Q)$
1c. $((P \rightarrow Q) \rightarrow (P \vee Q))$

2a. $(P \rightarrow (P \rightarrow Q))$
2b. $(P \rightarrow (Q \rightarrow P))$
2c. $(P \rightarrow (Q \rightarrow Q))$

3a. $(P \rightarrow \sim(P \wedge Q))$
3b. $(\sim P \rightarrow \sim(P \wedge Q))$
3c. $(P \rightarrow (P \wedge \sim Q))$

4a. $((P \rightarrow R) \wedge (Q \rightarrow R))$
4b. $((P \wedge Q) \rightarrow R)$
4c. $((P \vee Q) \rightarrow R)$

5a. $((P \rightarrow R) \vee (Q \rightarrow R))$
5b. $((P \wedge Q) \rightarrow R)$
5c. $((P \vee Q) \rightarrow R)$

6a. $((P \rightarrow Q) \vee (P \rightarrow R))$
6b. $(P \rightarrow (Q \wedge R))$
6c. $(P \rightarrow (Q \vee R))$

7a. $(P \rightarrow (Q \rightarrow R))$
7b. $((P \rightarrow Q) \rightarrow R)$
7c. $((P \wedge Q) \rightarrow R)$

8a. $(P \rightarrow (Q \vee R))$
8b. $(\sim(Q \vee R) \rightarrow \sim P)$
8c. $((\sim Q \vee \sim R) \rightarrow \sim P)$

C. A Puzzle About Conditionals and Conjunctions. We have been counting "if… then" as a conditional phrase, translated by the arrow.

P: Rex is going.
Q: Barbie going

1. If Rex is going then Barbie's going. $(P \rightarrow Q)$

But note that "if… then" can appear accompanied by the words "too", "as well", or "also".

2. **If** Rex is going **then** Barbie's going **too**.
3. **If** Rex is going **then** Barbie's going **as well**.
4. **If** Rex is going **then** Barbie's **also** going.

We noted earlier that "too", "as well", and "also" typically accompany conjunction phrases.

5. The movie is entertaining, **and** it's informative **too**.
6. Kids will enjoy the movie, **but** adults will like it **as well**.
7. Kids will enjoy the movie, **but** adults will **also** like it.

That suggests that Sentences (2), (3), and (4) should be translated with a **conjunction as its right part** (meaning "If Rex is going then both he and Barbie are going").

2*. **If** Rex is going **then** Rex is going and Barbie's going **too**.
3*. **If** Rex is going **then** Rex is going and Barbie's going **as well**.
4*. **If** Rex is going **then** Rex is going and Barbie goes **as well**.

All of these sentences would be translated like so.

$(P \rightarrow \underline{(P \wedge Q)})$

Build **truth tables** for "$(P \rightarrow Q)$" and "$(P \rightarrow (P \wedge Q))$" to show why **there's no semantic reason to make this change** – so that, for purposes of truth and validity, we can continue translating sentences (2), (3), and (4) as simply "$(P \rightarrow Q)$".

D. Using your answer to **C** (above), explain why the following two sentences are bound to be **logically equivalent**.

1. If we're going to Hawaii, then we're not going to Hawaii.
2. If we're going to Hawaii, then we're going to Hawaii without going to Hawaii.

E. **A Puzzle about Conditionals and Negated Disjunctions.** Notice that when an "if… then" sentence has a negated antecedent, we can sometimes use the words "not… either" or "neither" in the consequent.

P: Rex is going.
Q: Barbie going

1. If Rex isn't going then Barbie's not going. $(\sim P \rightarrow \sim Q)$

2. If Rex isn't going then Barbie's **not** going **either**.
3. If Rex isn't going then **neither** is Barbie.

Now, "neither" goes with "nor," and a "neither… nor" sentence is translated as the negation of an "either… or" sentence. That suggests that Sentences (2) and (3) should be translated with the **negation of a disjunction in the consequent** – as in the following sentence.

4. If Rex isn't going then **neither** he nor Barbie are going.

That would be translated like so.

$(\sim P \rightarrow \sim(P \vee Q))$

Build **truth tables** for "$(\sim P \rightarrow \sim Q)$" and "$(\sim P \rightarrow \sim(P \vee Q))$" to show why **there's no semantic reason to make this change** – so that, for purposes of truth and validity, we can continue translating sentences (2) and (3) as simply "$(\sim P \rightarrow \sim Q)$".

F. By which Chapter Two equivalence could we reduce the problem in **E** to the problem in **C**?